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Unique identification of graviton exchange effects in e^+e^- collisions*

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Abstract: Many types of new physics can lead to contact interaction-like modifications in e^+e^- processes below direct production threshold. We examine the possibility of uniquely identifying the effects of graviton exchange, which are anticipated in many extra dimensional theories, from amongst this large set of models by using the moments of the angular distribution of the final state particles. In the case of the $e^+e^- \to f\bar{f}$ process we demonstrate that this technique allows for the unique identification of the graviton exchange signature at the 5σ level for mass scales as high as $6\sqrt{s}$. The extension of this method to the $e^+e^- \to W^+W^-$ process is also discussed.

Keywords: Extra Large Dimensions, Field Theories in Higher Dimensions.

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Contents

1. Introduction

It is generally expected that new physics beyond the Standard Model(SM) will manifest itself at future colliders that probe the TeV scale such as the LHC and the Linear Collider. This new physics(NP) may appear either directly, as in the case of new particle production, e.g. , SUSY, or indirectly through deviations from the predictions of the SM. In the case of direct production, the discovery and identification of the NP would be relatively straightforward once masses and various couplings were determined through precision measurements. In the case of indirect discovery the effects may be subtle and many different NP scenarios may lead to the same or similar experimental signatures. Clearly, identifying the origin of the NP in these circumstances will prove more difficult and new tools must be available to deal with this potentiality.

Perhaps the most well known example of this indirect scenario in a collider context would be the observation of deviations in, e.g. , various e^+e^- cross sections due to apparent contact interactions [1]. There are many very different NP scenarios that predict new particle exchanges which can lead to contact interactions below direct production threshold; a partial list of known candidates is: a Z' from an extended gauge model [2, 3], scalar or vector leptoquarks [2, 4], R-parity violating sneutrino($\tilde{\nu}$) exchange [5], scalar or vector bileptons [6], graviton Kaluza-Klein(KK) towers [7, 8] in extra dimensional models [9, 10], gauge boson KK towers [11, 8], and even string excitations [12]. Of course, there may be many other sources of contact interactions from NP models as yet undiscovered, as was low-scale gravity only a few years ago.

If contact interaction effects are observed one can always try to fit the shifts in the observables to each one of the set of known theories and see which gives the best fit-an intensive approach followed by Pasztor and Perelstein [13]. Identifying the model that fits best, it may then be possible to select a starting point for further exploration and model building. Alternatively, it may be useful to devise a test or set of tests which will rather quickly divide the full set of all possible models into subclasses which can then be studied further by other techniques. In particular, it would be useful to have a method that rapidly identifies basic features about certain model classes. In this paper we propose such a technique that makes use of the specific modifications in angular distributions induced by $s-$ and $t-$channel exchanges of particles of various spins. As we will see below this method offers a way to uniquely identify the spin-2 aspects of graviton KK tower exchange (or, indeed, $\tilde{\nu}$ or any other possible spin-0 exchange) provided it is dominant the source of the new contact interaction. The *universality* of this interaction, that it is present for all possible final states, then uniquely singles out graviton exchange.

2. Technique

In order to introduce our technique, let us consider the normalized cross section for the process $e^+e^- \to f\bar{f}$ in the SM assuming $m_f = 0$ and $f \neq e$ for simplicity. This can be written as

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \frac{3}{8}(1+z^2) + A_{FB}(s)z\,, \tag{2.1}$$

where $z = \cos\theta$ and $A_{FB}(s)$ is the Forward-Backward Asymmetry which depends upon the electroweak quantum numbers of the fermion, f, as well as the center of mass energy of the collision, $\sqrt{s}$. This structure is particularly interesting in that it is equally valid for a wide variety of New Physics models: composite-like contact interactions, Z' models, TeV-scale KK gauge bosons, as well as for $t-$ or $u-$ channel leptoquark and bilepton exchanges at leading order in s/M^2, where M is the leptoquark or bilepton mass. In fact, for this observable the *only* deviation from the SM for any of these models will be through the variations in the value of $A_{FB}(s)$ since we have chosen to normalize the cross section.

Now let us consider taking moments of the normalized cross section above with respect to the Legendre Polynomials, $P_n(z)$. This can be done easily by re-writing eq. (2.1) as

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \frac{1}{2}P_0 + \frac{1}{4}P_2 + A_{FB}(s)P_1\,, \tag{2.2}$$

and recalling that the $P_n(z)$ are normalized as

$$\int_{-1}^{1} dz P_n(z)P_m(z) = \frac{2}{2n+1}\delta_{nm}\,. \tag{2.3}$$

Denoting such moments as $\langle P_n\rangle$, one finds that $\langle P_1\rangle = 2A_{FB}/3$, $\langle P_2\rangle = 1/10$ and $\langle P_{n>2}\rangle = 0$. In addition we also trivially obtain that $\langle P_0\rangle = 1$ since we have normalized the distribution so that this moment carries no new information. Thus, very naively, if we find that the $\langle P_{n>1}\rangle$ are given by their SM values while $\langle P_1\rangle$ differs from its corresponding SM value we could conclude that the NP is most likely one of those listed immediately above. If *both* $\langle P_{1,2}\rangle$ differ from their SM expectations while the $\langle P_{n>2}\rangle$ remain zero the source can only be $\tilde{\nu}$, or more generally, a spin-0 exchange in the $s-$channel. As we will see below only $s-$channel KK graviton exchange, since it is spin-2, leads to non-zero values of $\langle P_{3,4}\rangle$ while the $\langle P_{n>4}\rangle$ still remain zero. Of course the values of $\langle P_{1,2}\rangle$ will also be different from their SM values in this case but as we have just observed this signal is not unique to gravity. This observation seems to yield a rather simple test for the exchange of graviton KK towers.[1]

[1]The possibility of using a Fourier-type analysis in the case of the $e^+e^- \to \gamma+$ missing energy process to uniquely look for graviton tower emission was first discussed by [14].

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$N_{\rm bins}$	$\langle P_2\rangle(10^{-2})$	$\langle P_4\rangle(10^{-3})$	$\langle P_1\rangle$	$\langle P_3\rangle(10^{-3})$
10	9.0040	−26.7585	0.66000	−23.1000
20	9.7503	−6.8285	0.66500	−5.8188
50	9.9600	−1.0988	0.66640	−0.9330
200	9.9975	−0.0687	0.66665	−0.0583
1000	9.9999	−0.0027	0.66667	−0.0023
∞	10.0	0.0	2/3	0.0

Table 1: Dependence on $N_{\rm bins}$ for the first four moments of the normalized cross section appearing on the right hand side of eq. (2.1). Both $\langle P_{1,3}\rangle$ are in units of A_{FB}.

It is important to note that we could not have performed this simple analysis for the case of Bhabha scattering, i.e. , $e^+e^- \to e^+e^-$, as it involves both $s-$ and $t-$channel exchanges in the SM and thus *all* of the $\langle P_n\rangle$ would be non-zero.

Of course, the real world is not so simple as the idealized case we have just discussed for several reasons. First, we have assumed that we know the cross section precisely at all values of z, i.e. , we have infinite statistics with no angular binning. Secondly, to use the orthonormality conditions above we need to have complete angular coverage, i.e. , no holes for the beam pipe, etc. . To get a feeling for how important these effects can be let us first consider dividing the distribution into a finite number of angular bins, $N_{\rm bins}$, of common size $\Delta z = 2/N_{\rm bins}$. Instead of doing a simple integral we must perform a sum, i.e. , we make the replacement

$$\int_{-1}^{1} dz P_n(z)\frac{1}{\sigma}\frac{d\sigma}{dz} \to \sum_{\rm bins} \frac{P_n(z_i)\sigma_i}{\sigma}\,, \tag{2.4}$$

where i labels the bin number, σ_i is the cross section in each bin obtained by direct integration and z_i is the bin center at which the P_n are to be evaluated. The results of this analysis are shown in table 1 for the case of large statistics; here we see that as the number of bins grows large we rapidly recover the continuum results discussed above. Of course in any realistic experimental situation, $N_{\rm bins}$ remains finite but we see that a value of order 20 is reasonable as it strikes a respectable balance between the realistic demands of statistics, angular resolution and taking $N_{\rm bins}$ sufficiently large. The fact that we do not recover the trivial SM results above in this case can be considered as a 'background' in a loose sense. We will return to this point below.

Now let us assume that $N_{\rm bins} = 20$ and examine the effects of the necessary cut at small angles due to the beam pipe, etc. (Of course this cut is made symmetrically near both 0^o and 180^o so as to not induce additional backgrounds into the moments.) This is straightforward to implement from the above and leads to the results shown in table 2 for various values of the small angle cut. We note that by including this cut the value of z_i at which the P_n are evaluated changes for the two bins nearest the beam pipe at either end of the detector as we always assume they are to be evaluated at the center of the relevant range in z. Here we observe that the 'background contamination' of the naive SM result increases quite rapidly as we make the angular cut stronger.

Cut(mr)	$\langle P_2\rangle(10^{-2})$	$\langle P_4\rangle(10^{-3})$	$\langle P_1\rangle$	$\langle P_3\rangle(10^{-3})$
0	9.7503	−6.8285	0.66500	−5.8188
10	9.7428	−6.8981	0.66490	−5.9156
50	9.5652	−8.5590	0.66251	−8.2301
100	9.0159	−13.616	0.65508	−15.341
200	6.9030	−31.895	0.62600	−41.974

Table 2: Dependence on the cut at small scattering angles in milliradions assuming $N_{\text{bins}} = 20$ for the first four moments of the normalized cross section appearing on the right hand side of eq. (2.1). Both $\langle P_{1,3}\rangle$ are in units of A_{FB}.

What this brief study indicates is that for a realistic detector at a linear collider the simple and naive expectations for the various moments will receive 'backgrounds' that will need to be dealt with and subtracted from the real data to obtain information on the $\langle P_n\rangle$. In the real world these backgrounds can be found for a given detector through a detailed Monte Carlo(MC) study whose results will be influenced the detector geometry and by how well the properties of the detector are known. For our numerical analysis below we will follow a simpler approach by calculating the moments in the SM (after binning and cuts are applied) and then subtracting them from those obtained when the NP is present. In a more realistic analysis this means that we will assume that the detailed detector MC study can determine these backgrounds with reasonably high precision so that they can be subtracted once the actual data is available for analysis.

3. Analysis

Given the discussion above it is clear that we should begin by examining the process $e^+e^- \to f\bar{f}$; we will return to other potentially interesting processes below. To be specific we will concentrate on the model of Arkani-Hamed et al. [9], ADD, though our results are easily extended to the case of the the Randall-Sundrum model [10] below the graviton resonance production threshold. The differential cross section for $e^+e^- \to f\bar{f}$, now including graviton tower exchange, for massless fermions can be written as [7]

$$\begin{aligned}\frac{d\sigma}{dz} = N_c\frac{\pi\alpha^2}{s}\Bigg\{ \tilde{P}_{ij}\left[\frac{A^e_{ij}A^f_{ij}(2P_0+P_2)}{3} + 2B^e_{ij}B^f_{ij}P_1\right] - \\ -\frac{\lambda s^2}{2\pi\alpha\Lambda_H^4}\tilde{P}_i\left[\frac{v^e_i v^f_i\ (2P_3+3P_1)}{5} + a^e_i a^f_i P_2\right] + \\ +\frac{\lambda^2 s^4}{16\pi^2\alpha^2\Lambda_H^8}\left[\frac{(16P_4+5P_2+14P_0)}{35}\right]\Bigg\}, \end{aligned} \tag{3.1}$$

where the indices i,j are summed over the γ and Z exchanges, $z = \cos\theta$ as above, $\tilde{P}_{ij}$ and $\tilde{P}_i$ are the usual dimensionless propagator factors (defined in e.g. , [2]), $A^f_{ij} = (v^f_i v^f_j + a^f_i a^f_j)\,, B^f_{ij} = (v^f_i a^f_j + v^f_j a^f_i)\,, P_n = P_n(z)$ and N_c represents the number of colors of the final state. Λ_H is the cutoff scale employed by Hewett [7] in evaluating the summation over the tower of KK graviton propagators and $\lambda = 1$ will be assumed in what follows. Our results will not depend upon this particular choice of sign. In this expression we explicitly see the dependence on the $P_{n>2}$ associated with the exchange of the tower of KK gravitons. Note that term proportional to P_3 occurs in the interference between the SM and gravitational contributions whereas the term proportional to P_4 occurs only in the pure gravity piece. This implies that for $\sqrt{s} \ll \Lambda_H$ it will be $\langle P_3 \rangle$ which will show the largest shifts from the expectations of the SM. With the polarized beams that we expect to have available at a linear collider, a z-dependent Left-Right Asymmetry, A_{LR}, can also be formed and provides an additional observable; this is proportional to the difference $d\sigma_L - d\sigma_R$ with $\sigma_{L,R}$ being the cross section obtained with left- or right-handed polarized electrons. Using the notation above this asymmetry can be written as

$$A_{LR}(z) = \frac{\tilde{P}_{ij}\left[B^e_{ij}A^f_{ij}(P_2+2P_0)/3 + A^e_{ij}B^f_{ij}P_1\right]}{D} - \\ - \frac{\lambda s^2}{2\pi\alpha\Lambda_H^4}\frac{\tilde{P}_i\left[a^e_i v^f_i\ (2P_3+3P_1)/5 + v^e_i a^f_i P_2\right]}{D}, \tag{3.2}$$

where D is given by the curly bracket in the cross section expression above. Note that in the presence of graviton exchange this quantity also explicitly depends upon the $P_{n>2}$ with the leading corrections again expected in $\langle P_3 \rangle$.

Our approach will be as follows: we consider two observables (i) the normalized unpolarized cross section and (ii) the normalized difference of the polarized cross sections $\sim (d\sigma_L - d\sigma_R)/dz$, which is essentially given by the numerator terms in the expression for A_{LR}. We then calculate the first four non-trivial moments of these two observables for the μ, τ, b, c and t final states within the SM including the effects of Initial State Radiation(ISR). (Note that for the $t\bar{t}$ final state we need to generalize the expressions above to include finite mass effects. This means that for $t\bar{t}$ all of the moments will become $\sqrt{s}$ dependent asymptoting to the values given above as $\sqrt{s} \to \infty$.) Here we will assume tagging efficiencies of 100%, 100%, 80%, 60% and 60%, respectively, for the various final states and that $N_{\text{bins}} = 20$ with θ_{cut} =50mr for purposes of demonstration. The resulting values for the $\langle P_n \rangle$ as calculated in the SM will be called 'background' values consistent with our discussion above. Next, we calculate the same moments in the ADD model by choosing a value for the parameter Λ_H. Combining both observables and summing over the various flavor final states we can form a χ^2 from the deviation of the $\langle P_{3,4} \rangle$ moments from their SM 'background' values. For a fixed integrated luminosity this can be done using the statistical errors as well as the systematic errors associated with the precision expected on the luminosity and polarization measurements. (Here we will assume the values $\delta L/L = 0.25\%$ and $\delta P/P = 0.3\%$ in order to incorporate these systematic effects.) Next we vary the value of the scale Λ_H until we obtain a 5σ deviation from the SM; we call this value of Λ_H the Identification Reach as it is the maximum value for the scale at which we observe a 5σ deviation from the SM values of $\langle P_{3,4} \rangle$ which we now know can only arise due to the effects of graviton exchange. Note that this value of the scale should *not* be confused with the Discovery Reach at which one observes an

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f	$\langle P_2\rangle(10^{-2})$	$\langle P_4\rangle(10^{-3})$	$\langle P_1\rangle$	$\langle P_3\rangle(10^{-3})$
μ,τ	9.58	-8.58	0.319	-3.97
b	9.58	-8.58	0.419	-5.21
c	9.58	-8.58	0.407	-5.06
t	4.41	-6.87	0.269	-3.34

Table 3: SM values for the various moments of the normalized unpolarized cross section for various flavors as $\sqrt{s} = 500\,\text{GeV}$. Only the top quark is assumed massive. We take $N_{\text{bins}} = 20$ and $\theta_{\text{cut}} = 50\text{mr}$.

f	$\langle P_2\rangle(10^{-2})$	$\langle P_4\rangle(10^{-3})$	$\langle P_1\rangle$	$\langle P_3\rangle(10^{-3})$
μ,τ	8.41	-8.03	0.286	-12.69
b	5.41	-6.65	0.376	-16.04
c	11.76	-9.00	0.448	5.93
t	5.45	-7.14	0.283	3.92

Table 4: Same as the previous table but now assuming $\Lambda_H = 2\,\text{TeV}$.

overall deviation from the SM. Here we are specifically looking only at deviations in these special moments since they alone are graviton sensitive. Although both the $\langle P_{1,2}\rangle$ *also* deviate from their SM values these shifts cannot be directly attributed to a spin-2 exchange. (As noted previously, the shift in $\langle P_1\rangle$ results in any of the NP models listed above whereas a shift in $\langle P_2\rangle$ occurs whenever the new s-channel exchange is *not* spin-1, e.g. , $\tilde{\nu}$ exchange.)

To first get an idea of the influence of the graviton KK exchange consider the results shown in tables 3 and 4. The first of these tables shows the moment values in the SM for several final state flavors at a $\sqrt{s} = 500\,\text{GeV}$ collider. Note that the background value of $\langle P_3\rangle$ is proportional to $\langle P_1\rangle$ for each flavor due to 'leakage' and that the moments for the top quark differ significantly from the others due to the large finite mass effects. The proportionality of $\langle P_3\rangle$ and $\langle P_1\rangle$ for all flavors signals that $\langle P_3\rangle \neq 0$ is arising from the background and not from NP sources. In the second table we see that when the graviton KK contributions are turned on there are respectable shifts in $\langle P_{1,2}\rangle$ for all flavors of order $10-30\%$ while the corresponding shifts in $\langle P_4\rangle$ are somewhat smaller. On the otherhand the deviations in $\langle P_3\rangle$ are at the $100-400\%$ level and even changes in sign are observed. As expected, $\langle P_3\rangle$ shows the greatest sensitivity to graviton exchange. Note that the values of $\langle P_3\rangle$ and $\langle P_1\rangle$ are no longer correlated for the different flavors as is the case for the SM background. Clearly the large shifts in $\langle P_3\rangle$ and the fact that the values of $\langle P_3\rangle/\langle P_1\rangle$ are now flavor dependent and differ significantly from their SM values is a unique signature for KK graviton exchange.

Returning to our calculation outlined above we can straightforwardly determine the ID reach; this is shown in figure 1 for several values of $\sqrt{s}$ as a function of the integrated luminosity. Specifically, for a $\sqrt{s} = 500\,\text{GeV}$ machine with an integrated luminosity of $1\ ab^{-1}$ the ID reach with single(double) beam polarization is found to be 2.6(3.0) TeV, i.e. , $(5-6)\sqrt{s}$. We remind the reader that the corresponding search reach for these luminosities is in range of $(9-10)\sqrt{s}$ [15]. Note that the ID reach obtained by this approach is a rather respectable fraction, more than half, of the search reach.

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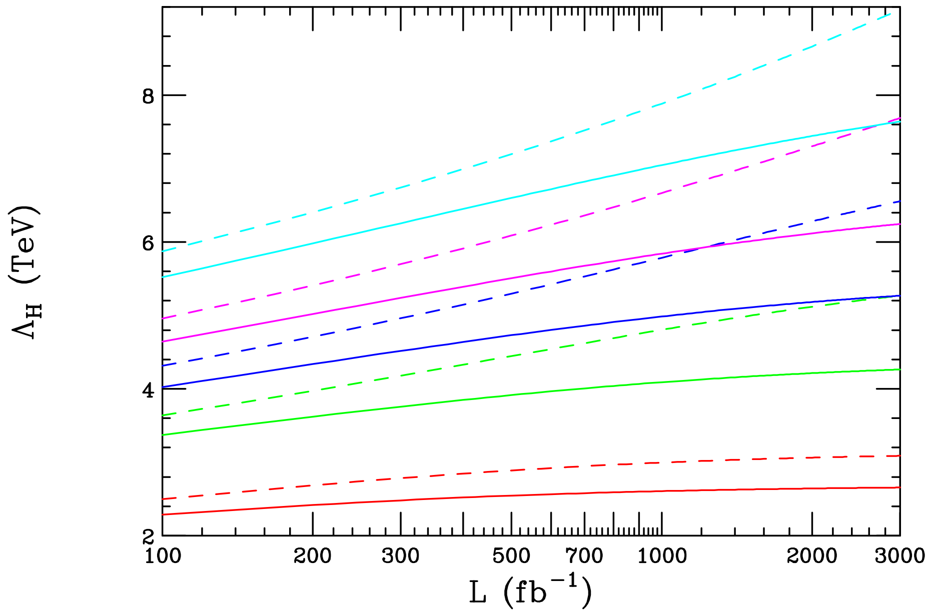

Figure 1: Identification reach in Λ_H as a function of the integrated luminosity fro the process $e^+e^- \to f\bar{f}$, with f summed over μ, τ, b, c and t. The solid(dashed) curves are for an e^- polarization of 80%(together with a e^+ polarization of 60%). From bottom to top the pairs of curves are for $\sqrt{s} = 0.5, 0.8, 1, 1.2$ and $1.5\,\text{TeV}$, respectively.

Before turning to other possible processes, it is instructive to examine what influence other NP scenarios might have on the $\langle P_n \rangle$. An artificial scenario that would most closely mimic gravity would be the $s-$channel exchange of a universally coupled scalar field. Like gravity, since the scalar exchange does not interfere with the γ and Z SM contributions, the leading effects are effectively from dimension-8 operators. To be specific, we assume $\sqrt{s} = 500\,\text{GeV}$, a Yukawa coupling of electromagnetic strength and a scalar mass of 1.1 TeV, a value chosen so as to produce sizeable shifts in $\langle P_2 \rangle$. This toy model yields the results shown in table 5 where reasonable shifts in all the $\langle P_n \rangle$ are apparent. In comparison to graviton KK graviton exchange, several differences with the scalar exchange case are immediately obvious: (i) the shifts in both $\langle P_1 \rangle$ and $\langle P_4 \rangle$ are larger in the scalar case whereas those for $\langle P_3 \rangle$ are significantly smaller by almost an order of magnitude. (ii) While both $\langle P_{1,3} \rangle$ are shifted, their *ratio* remains at its SM value for all flavors unlike in the case of gravity. It is clear that even this ad hoc toy model will not be confused with graviton KK tower exchange.

4. $e^+e^- \to W^+W^-$

Can we use other processes to uniquely isolate the effects of graviton KK tower exchange? The other SM processes with large tree-level cross sections in which gravitons can be exchanged are $e^+e^- \to e^+e^-, \gamma\gamma, ZZ$, and W^+W^- all of which involve $t-$ and/or $u-$channel exchanges. This would apparently disqualify them from further consideration. The W^+W^- case is, however, special [16] because ideally the offending $t-$channel ν exchange can be

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f	$\langle P_2\rangle(10^{-2})$	$\langle P_4\rangle(10^{-3})$	$\langle P_1\rangle$	$\langle P_3\rangle(10^{-3})$
μ,τ	7.03	-7.55	0.236	-2.93
b	4.23	-6.41	0.188	-2.34
c	5.78	-7.04	0.248	-3.08
t	2.35	-5.87	0.148	-1.83

Table 5: Same as the previous table but now assuming the $s-$channel exchange of a 1.1 TeV scalar with universal couplings to all fermions as described in the text.

removed through the use of right-handed beam polarization leaving only the $s-$channel γ, Z exchanges. The remaining purely right-handed SM cross section is only quadratic in z and this will not change if, e.g. , Z' or new $s-$channel scalar contributions are also present. As we will discuss in detail below the difficulty in this case is that the left-handed cross section is much larger than the right-handed one so that the possibility of 'contamination' from the wrong polarization state is difficult to eradicate unless very good control over the beam polarization is maintained. Apart from the problem of isolating the purely right-handed part of the cross section we might again conclude that the non-zero $\langle P_{3,4}\rangle$ moments will be be a unique signature of graviton exchange for this process. Futhermore, since the pure gauge sector of the SM individually conserves C and P, there are no terms in the cross section linear in z and thus $\langle P_1\rangle$ is expected to be zero in the SM and in many other NP extensions. Such terms are, however, generated by KK graviton exchange so that a non-zero value of $\langle P_1\rangle$ is also a potential gravity probe.

Apart from new particle exchanges there is another source of NP that can modify the right-handed W-pair cross-section in a manner similar to gravity and would most likely be observed in lowest order as a dimension-8 operator (as is graviton KK tower exchange): anomalous gauge couplings (AGC) [17]. As is well known AGC can be C and/or P violating; one that violates both C and P but is CP conserving can (and does) produce non-zero $\langle P_{1,3,4}\rangle$ moments. Decomposing the WWV ($V = \gamma, Z$) vertex in the most general way allowed by electromagnetic gauge invariance yields 7 different anomalous couplings for each V with the corresponding form factors denoted by f_i^V. (When weighted by the sum over the γ and Z propagators in $e^+e^- \to W^+W^-$ these form factors are sometimes written as F_i, only two of which, $F_{1,3}$, are non-zero at the tree-level in the SM.) There is a single term in this general vertex expression with the correct C and P properties: that proportional to $f_5^V\epsilon^{\mu\alpha\beta\rho}(q^- - q^+)_\rho\epsilon_\alpha(W^-)\epsilon_\beta(W^+)$ with $q^\pm$ the outgoing $W^\pm$ boson momenta, the ϵ's are their corresponding polarization vectors and f_5^V being the relevant form factor. We note that even in the SM, though absent at the tree-level, this term is generated at one-loop from the usual fermion triangle anomaly graph. As we will see such a term will generate non-zero values for all of $\langle P_{1,3,4}\rangle$.

It appears that the possibility of non-zero AGC would contaminate our search for unique graviton exchange signatures. There is a way out of this dilemma; while gravity induces non-zero values for all of the $\langle P_{1,3,4}\rangle$ from the angular distributions for $e^+e^-_R \to W^+W^-$ *independently* of the final state W polarizations, the $f_5^V \neq 0$ (i.e. , F_5) couplings

	$\langle P_2\rangle(10^{-1})$	$\langle P_4\rangle(10^{-3})$	$\langle P_1\rangle(10^{-2})$	$\langle P_3\rangle(10^{-2})$
SM	-2.14	-6.94	0.0	0.0
$\Lambda_H = 2\,\mathrm{TeV}$	-2.16	-8.88	-4.88	2.46
SM'	1.05	-113.30	44.24	-13.07
SM"	-1.11	-41.33	14.30	-4.22

Table 6: Moments of the normalized W pair production cross section assuming purely right-handed electrons and isolating the $TT+LL$ final states at $\sqrt{s} = 500\,\mathrm{GeV}$. An angular cut $|\cos\theta| \leq 0.9$ has been applied. The SM prediction is compared with that for KK graviton tower exchange. Also shown is the SM prediction, labelled by SM'(SM") for the case of 80% right-handed e^- and 60% left-handed e^+ polarization(both beams with 90% polarization.)

only contribute to the final state with mixed polarizations, i.e. , transverse plus longitudinal, $TL+LT$. We recall that by measuring the angular distribution of the decaying W relative to its direction of motion we can determine its state of polarization; here we do not differentiate the two possible states of transverse polarization. Writing

$$\frac{d\sigma_R}{dz} \sim \Sigma_{TT} + \Sigma_{LL} + \Sigma_{TL+LT}\,, \tag{4.1}$$

in the absence of CP violation there are only 4 relevant F_i and one finds

$$\begin{aligned} \Sigma_{TT} &\sim F_1^2(1-z^2) \\ \Sigma_{LL} &\sim \left[F_3 - \left(1 - \frac{2m_W^2}{s}\right)F_1 + \frac{\beta^2 s}{2m_W^2}F_2\right]^2 (1-z^2) \\ \Sigma_{TL+LT} &\sim (F_3+\beta z F_5)^2(1+z^2) + 2F_5(F_3+\beta z F_5)z(1-z^2) + F_5^2(1-z^2)^2\,, \end{aligned} \tag{4.2}$$

where β is the speed of the outgoing W's. Here we see that in the SM both the TT and LL terms are proportional to $1-z^2$ while the $TL+LT$ term is proportional to $1+z^2$; no terms linear in z are present. A non-zero F_5 induces additional terms in the case of the $TL+LT$ final state which now contains linear, cubic and quartic powers of z similar to that generated by gravity. However, the TT and LL final states receive no such contributions. Thus observing non-zero values of $\langle P_{1,3,4}\rangle$ (again, above backgrounds) for W pairs in the $TT+LL$ final states produced by right-handed electrons *is* a signal for KK graviton tower exchange. Numerically the TT fraction will dominate in the energy region of interest to us below.

To get an idea of the size of the graviton contributions to the $TT+LL$ part of the right-handed cross section, we show in table 6 a comparison of the $\langle P_n\rangle$ obtained in the SM and in the case with graviton KK tower exchange assuming $\Lambda_H = 2\,\mathrm{TeV}$ and $\sqrt{s} = 500\,\mathrm{GeV}$. For later purposes, a cut of $|\cos\theta| \leq 0.9$ has been applied and an efficiency of 40% has been assumed to both reconstruct the two W's and to isolate the $TT+LL$ final state through their decay angular distribution. ISR has been ignored in these results. Here we see that the shifts for $\langle P_{1,3}\rangle$ are quite large while those for $\langle P_2\rangle$ are somewhat smaller as would be expected from the interference of the SM and KK gravity contributions. Provided purely right-handed electron beams were available, we can easily determine the graviton KK tower

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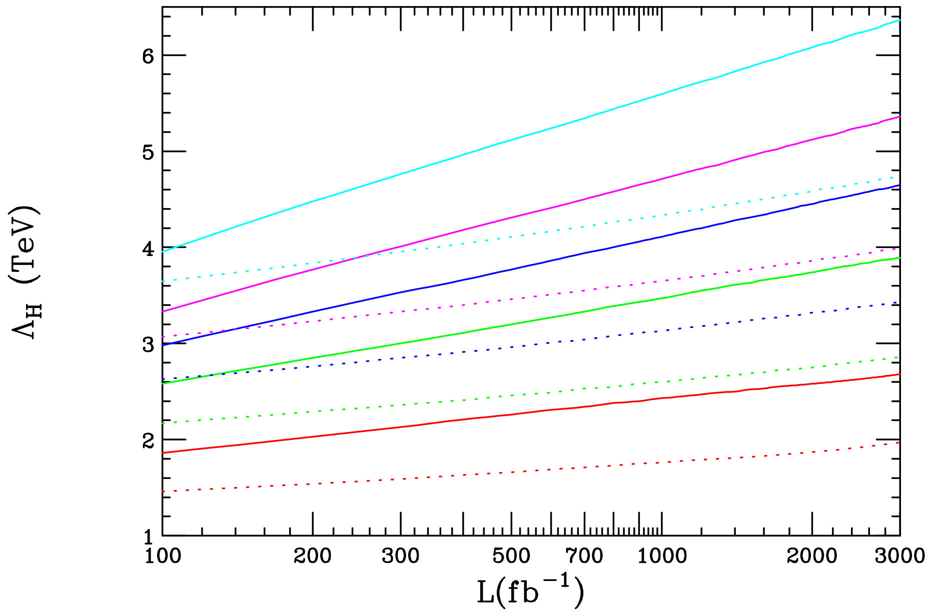

Figure 2: Identification reach in Λ_H as a function of the integrated luminosity for the process $e^+e^-_R \to W^+W^-$(solid), using only the $TT + LL$ final states. From bottom to top the curves are for $\sqrt{s} = 0.5, 0.8, 1, 1.2$ and 1.5 TeV, respectively. The corresponding dotted curves are for the case of 80% e^- and 60% e^+ polarization.

ID limit in this case. (Recall that we can now make use of all three of the moments $\langle P_{1,3,4}\rangle$ in the χ^2.) The results can be found in figure 2. Note that the identification reach found in this extremely idealized situation of 100% right-handed beam polarization is somewhat inferior to that found in the case of the $f\bar{f}$ final state, i.e. , $\sim (4-5)\sqrt{s}$ at best. In any more realistic situation, especially when systematic effects are included, we expect this ID reach to significantly degrade.

Perhaps, the closest we may be able to come to this idealized case experimentally it to assume double beam polarization taking the initial $e^-(e^+)$ as right(left)-handed as possible. Assuming polarizations of 80% and 60% for the e^- and e^+ beams, respectively, increases the right-handed part of the cross section by a factor of 2.88 while reducing the left-handed part by a factor of 0.08. Unfortunately, the left-handed piece containing the t-channel ν exchange diagram is very large even after the cut of $|\cos\theta| \leq 0.9$ has been applied; recall that the left-handed part of the cross section has a large forward peak due to the $t-$channel exchange. Table 6 shows that with this degree of polarization (and even if both polarizations are unrealistically greater) the amount of contamination from the left-handed cross section is so large that making any claim to a unique identification of graviton exchange would be quite tenuous. We can still ask the same question as above: at what value of Λ_H do the shifts in the $\langle P_n\rangle$ correspond to a 5σ deviation from the SM? The answer shown in figure 2, however, is no longer a unique identification of gravity but only a signal for the clear discovery of NP. Once contaminated by the left-handed cross section, other sources of NP, such as a Z', can also lead to identical changes in the values of the moments.

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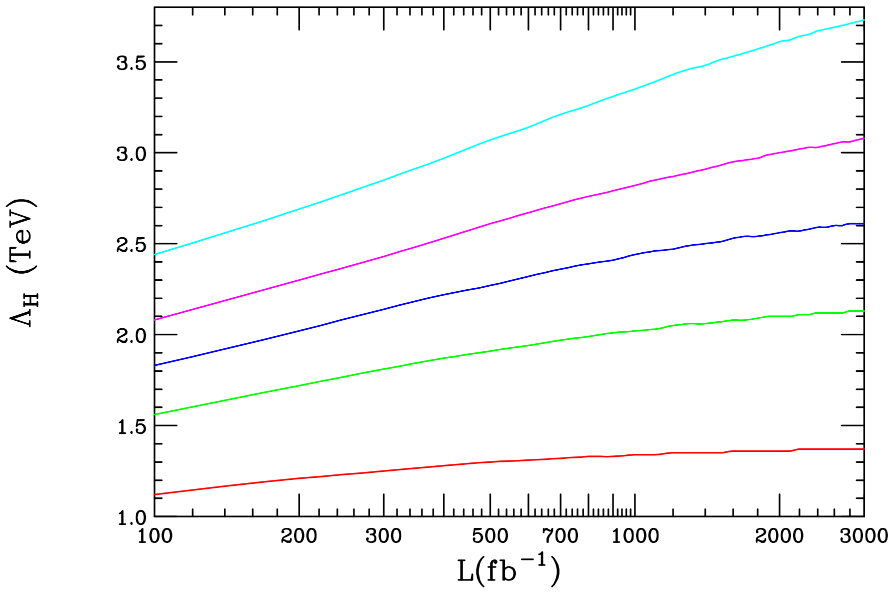

Figure 3: Same as in the previous figure but now using the combined analysis as discussed in the text.

The last possibility for salvaging this situation is to measure the W^+W^- cross section with two or more sets of different beam polarizations and then attempting to extract the purely right-handed piece from these measurements, again keeping only the $TT + LL$ contributions. In the case of two polarized beams this is perhaps best demonstrated by examining what happens when we combine two sets of data: one with $P(e^-) = -80\%$ and $P(e^+) = 60\%$ and the other with both polarizations flipped. (This is not necessarily the optimized choice of polarizations.) In comparison to the idealized purely right-handed case discussed above, here we suffer from having to be able to very precisely subtract the additional large backgrounds arising from the left-handed parts of the cross section. In addition, both reduced statistics (since the luminosity is divided between both measurements) and the systematic errors associated with the polarization uncertainties will lead to further reductions in the anticipated identification reach. In fact, assuming polarization uncertainties of $\delta P/P = 0.3\%$ as above, we might expect these systematic effects to play an important role in the measurement error budget.

Figure 3 shows the results of this analysis. Here we see that, as expected, the identification reach at large luminosities saturates due to the size of the systematic errors in extracting the right-handed piece of the cross section. The 5σ identification reach is found to be roughly $\sim 2.5\sqrt{s}$ for integrated luminosities of order 1 ab^{-1} which is far below that found for fermion pairs and the naive estimate we obtained in the case of purely right-handed e^- polarization. It is unlikely that a more judicious choice of beam polarizations could drastically increase this reach and make it competitive with that found for the $f\bar{f}$ final state.

5. Summary and conclusion

Many new physics scenarios predict the existence of contact interaction-like deviations from SM cross sections at high energy e^+e^- colliders. If such effects are observed our next task would be to identify their origin. In this paper we have suggested a technique by which the deviations induced by the spin-2 aspects of exchange of KK gravitons can be uniquely isolated from all other possible sources by an examination of the angular moments of the polarization dependent cross sections employing Legendre polynomials. The universality of these new interactions, associated with all possible final states, then uniquely picks out graviton exchange as the source for the deviaions observed. The technique is applicable when the 'background' SM process proceeds only through $s-$channel exchanges or when it can be made to do so by a special choice of beam polarization(s). The canonical process to study is $e^+e^- \rightarrow f\bar{f}$ for $f \neq e$. In this case it was found that by combining several final states the graviton exchange contributions can be uniquely identified at the 5σ level for ADD mass scales as large as $(5-6)\sqrt{s}$ in the Hewett scheme. This compares rather favorably to the search reach of $(9-10)\sqrt{s}$ using this same process. The reaction $e^-_R e^+ \rightarrow W^+W^-$ for the TT and LL final states also proceeds only via $s-$channel exchange in the SM and can also be used to obtain a unique signature for graviton exchange. The $LT+TL$ modes, which we do not include, were shown to be capable of receiving graviton-like contributions from the C and P odd, CP even anomalous trilinear couplings f_5^V. The difficulty in the case of the W^+W^- final state is that 100% polarized beams do not exist so that measurements made with different beam polarizations must be combined to extract the values of $d\sigma_R$ and as such, will suffer from sizeable systematic uncertainties. Though no attempt was made to optimize the choices of beam polarization, it was shown that even with both beams polarized the identification reach in this channel is somewhat below $\sim 2.5\sqrt{s}$ which is less than half that found for the $f\bar{f}$ final state. It is unlikely that optimization can lead to any sizeable improvement of this identification reach.

Hopefully the effect of new contact interactions will be observed at the Linear Collider so that these new techniques can be employed.

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